strength exactly without strengthening the excitation or without amplifying the signal. Using a high-gain amplifier after the detection made the measurement confusing because the leakage microwave power interfering with the signal was also amplified. But it is clear that the dc field range of surface magnetostatic waves becomes narrower as the gap increases.

CONCLUSION

We have theoretically shown that the upper limit of the frequency spectrum of surface magnetostatic waves changes continuously from $\omega = \omega_0 + \omega_M$ to $\omega = \omega_0 + \omega_M/2$ and the cutoff wavenumber changes continuously from infinity to a certain value as the gap between the surfaces and metal walls changes from zero to infinity in the direction in which the magnetic potential decays exponentially. Experimental results obtained at X band agree qualitatively with the theoretical estimates. By using the above results, new types of tunable microwave filters using surface magnetostatic waves are possible. These filters are mechanically tunable in contrast with the usual YIG filters that are tuned by controlling the dc magnetic field.

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Waveguide Sandwich Filters

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***Abstract*—A new class of waveguide filters is introduced, constructed from several thin plates sandwiched together. The combination of alternate plates having large and small rectangular apertures leads to a propagating structure which possesses a bandstop response and prescribed characteristic impedance. This basic element may be used as a simple compact bandstop filter, particularly where the main passband and stopband are well separated, such as in harmonic rejection. For filters with many stopbands, combinations of several waveguide sandwich filter elements are used to provide the main passband and the required attenuation characteristics in the prescribed stopbands. Although the filter is ideally suited for bandstop filtering due to its small size, low cost, low loss, and high power handling capability, additional applications to bandpass filtering and dispersive delay line operation are also cited.**

INTRODUCTION

THE SIMPLEST form of the waveguide sandwich filter [1] is for use in a rectangular waveguide system supporting the dominant H_{01} mode. The laminar construction uses two different plates as shown in Fig. 1. The width a of the rectangular apertures is the same as the width of the waveguide into which the filter is to operate and the two types A and B have different heights b_1 and b_2. Normally, the plates will be of the same thickness to reduce the overall cost of construction.

The overall waveguide filter element is formed from a large number of plates sandwiched together with the plates type A and B alternating. Normally, a rigid assembly is formed by dowelling the plates together and then soldering from the outside. The basic section then formed is shown in Fig. 2, and is characterized by a waveguide of uniform width with periodic transverse slots in the broad walls. Thus, by using an equivalent circuit comprised of a cascade of n identical sections, each of which is formed from a series stub symmetrically embedded in a short piece of uniform waveguide, the overall electrical properties of this basic waveguide sandwich filter element may be deduced. The device is shown to be essentially a bandstop structure with a broad passband at a lower frequency, but above the normal waveguide cutoff frequency.

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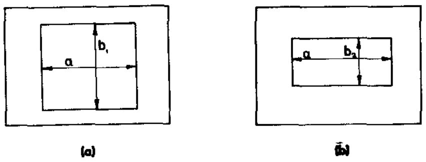

Fig. 1. Two types of plates used for the laminar construction.

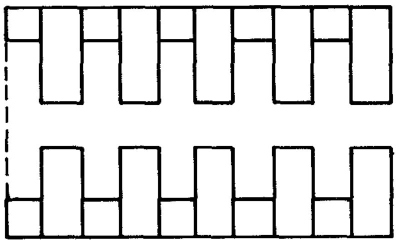

Fig. 2. Typical cross section of waveguide sandwich filter.

One of the main applications of the device is for use as a harmonic rejection filter where prescribed bands are to be rejected. For example, in a communications system operating in the 5.9–6.4-GHz band, normally spurious signals occur in the strict harmonic bands, i.e., 11.8–12.8 GHz, 17.7–19.2 GHz, etc., and consequently it will only be necessary to provide channel rejection in these limited frequency bands. One basic waveguide sandwich filter element may be used for each harmonic band and the several elements combined to provide a good passband. This may be contrasted to the conventional method of harmonic rejection using either waffle iron or corrugated waveguide filters [2] which suppress all frequencies up to a prescribed point. However, in general, those filters are much larger and more expensive than the sandwich filter. For the one band case, the single element may be readily tapped directly with holes to accommodate a standard waveguide flange from an adjacent component, thus resulting in a low-cost component. For several harmonic bands, a multistage filter is required and special care must be taken to reject the higher ordered modes which may be excited. Examples of the single and double band cases are presented.

Using the basic sandwich filter element, other types of filters may be constructed. An example of the design of a bandpass channel filter with inherent harmonic rejection is presented resulting in a low-cost compact structure. Finally, the use of the structure as a dispersive delay line is considered and possible applications mentioned.

THE BASIC SANDWICH FILTER ELEMENT

One decomposition of the sandwich filter element shown in Fig. 2 is into a cascade of identical sections, one of which is illustrated in Fig. 3. For each mode of the type H_{0n} an equivalent circuit may be constructed for the sections comprised of a series short-circuited stub symmetrically located in a uniform length of waveguide. Since the thickness of the plates used to form the stubs is considerably less than a guide wavelength for most operating frequencies, the assumption of a series connec-

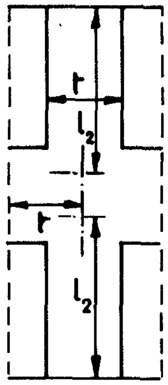

Fig. 3. Basic section.

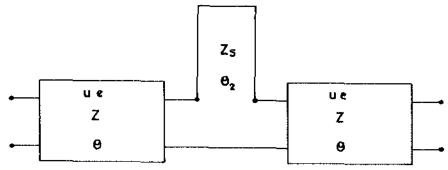

Fig. 4. Equivalent circuit of basic section.

tion will be valid. However, the reference plane for the location of the stub main guide connection must be taken at the plane of symmetry. Similarly, the effective length of the stub may be assumed to be the actual length plus the plate thickness, due to fringing field effects.[1] Thus the equivalent circuit for the section shown in Fig. 3 is illustrated in Fig. 4, where the overall transfer matrix for each length of line for the H_{01} mode is

$$\begin{bmatrix} \cos\theta & jZ\sin\theta \\ jY\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

with $Z = 1/Y$, the characteristic impedance normalized with respect to the terminating guide and

$$\theta = \beta t$$

where t is the thickness of the plates and β is the phase constant of the guide.

The series element possesses a reactance

$$X = 2Z_s \tan(\theta_2) \tag{2}$$

where $\theta_2 = \beta l_2$, l_2 being the effective length of the stubs, and

$$Z_s = \frac{Zt}{b_2} \tag{3}$$

where b_2 is the height of the main guide. [NB, the factor 2 occurring in (2) is due to the effect of slots being in both the upper and lower broad walls of the waveguide, resulting in the series connection of the two stubs and $b_1 = b_2 + 2l_2$.]

The overall transfer matrix of the basic section is therefore

[1] Deduced from experimental results on this type of structure. A correction term may be used for $b_2 < 2r$ [2].

$$\begin{bmatrix} \cos\theta & jZ\sin\theta \\ jY\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & j2Z_s\tan\theta_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & jZ\sin\theta \\ jY\sin\theta & \cos\theta \end{bmatrix}$$
$$= \begin{bmatrix} \cos 2\theta - Z_sY\tan\theta_2\sin 2\theta & j(Z\sin 2\theta + 2Z_s\tan\theta_2\cos^2\theta) \\ jY(\sin 2\theta - 2Z_sY\tan\theta_2\sin^2\theta) & \cos 2\theta - Z_sY\tan\theta_2\sin 2\theta \end{bmatrix} \tag{4}$$

which may be written as

$$\begin{bmatrix} \cos\psi & jZ_I\sin\psi \\ jY_I\sin\psi & \cos\psi \end{bmatrix} \tag{5}$$

where

$$\psi = \cos^{-1}\left[\cos 2\theta - Z_sY\tan\theta_2\sin 2\theta\right] \tag{6}$$

and

$$Z_I = Z\left[\frac{1 + Z_sY\tan\theta_2\cot\theta}{1 - Z_sY\tan\theta_2\tan\theta}\right]^{1/2} \tag{7}$$

is the image impedance of the section.

Substituting for Z_s, θ, and θ_2, we have

$$\psi = \cos^{-1}\left[\cos 2\beta t - \frac{t}{b_2}\tan\beta l_2\sin 2\beta t\right] \tag{8}$$

and

$$Z_I = Z\left[\frac{1 + (t/b_2)\tan\beta l_2\cot\beta t}{1 - (t/b_2)\tan\beta l_2\tan\beta t}\right]^{1/2}. \tag{9}$$

For most operating frequencies, βt will be small, and therefore, for $\beta l_2 < \pi/2$ we have

$$\psi \approx \cos^{-1}\left[1 - 2\beta^2t^2 - 2t^2\beta^2\frac{l_2}{b_2}\cdot A\right] \tag{10}$$

or

$$\psi \approx 2\sin^{-1}\left[t\beta\left(1 + \frac{l_2}{b_2}A\right)^{1/2}\right] \tag{11}$$

and

$$Z_I \approx Z\left[1 + \frac{l_2A}{b_2}\right]^{1/2} \tag{12}$$

where

$$A = \frac{\tan\beta l_2}{\beta l_2}. \tag{13}$$

In most design calculations, it will be necessary to determine the parameters of the section from the required stopband behavior and a prescribed impedance relative to the main guide in the passband. From the guide size and the location of the stopband ($\beta l_2 \approx \pi/2$), β and l_2 will be determined. If the height of the standard waveguide operating system is b_0, then for a prescribed characteristic impedance Z_1 normalized relative to the terminating guide, we have, from (12)

$$Z_1\frac{b_0}{b_2} = \left[1 + \frac{l_2A}{b_2}\right]^{1/2} \tag{14}$$

or

$$\frac{b_2}{b_0} = (Z_1^2 + K^2)^{1/2} + K \tag{15}$$

where

$$K = \frac{l_2A}{2b_0}. \tag{16}$$

To obtain the behavior of the waveguide sandwich filter element comprised of n of the basic sections, the transfer matrix (5) must be raised to the power n. Hence the transfer matrix of the basic element is

$$\begin{bmatrix} \cos\psi & jZ_I\sin\psi \\ jY_I\sin\psi & \cos\psi \end{bmatrix}^n = \begin{bmatrix} \cos n\psi & jZ_I\sin n\psi \\ jY_I\sin n\psi & \cos n\psi \end{bmatrix}. \tag{17}$$

At either end of the filter, theoretically we require a half thickness plate and an ideal transition into the standard guide. However, there is a shunt capacitance effect inherent in the transition, and therefore the use of a full thickness plate does not increase the capacitance effect by a large amount. In order to compensate for this shunt capacitance effect, series inductance is introduced; that is, an extra plate with a large aperture (type A). Thus a simple two stage section with these additional end plates is shown in Fig. 5, and the overall section still requires only two types of plates.

DESIGN OF A SINGLE BAND BANDSTOP FILTER

For the waveguide sandwich filter element described by the transfer matrix (17) operating into the normalized waveguide system, the insertion loss L is

$$L = 10\log\left[1 + \frac{(Z_I - Y_I)^2}{4}\sin^2 n\psi\right]. \tag{18}$$

In the rejection band, the series stubs are near to resonance and therefore,

$$\tan\theta_2\,|_{\theta_2\approx\pi/2} \approx \frac{1}{(\pi/2)(1 - \lambda g_0/\lambda g)} \tag{19}$$

where λg is the guide wavelength and λg_0 is the resonant frequency guide wavelength. Therefore, from (13),

$$A = \frac{4}{\pi^2(\lambda g/\lambda g_0 - 1)} \tag{20}$$

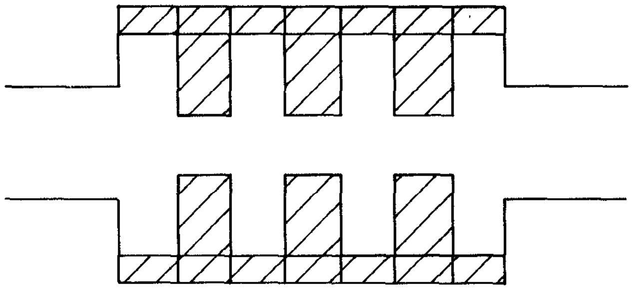

Fig. 5. Second ordered section illustrating the terminations.

and (11) becomes

$$\psi = 2 \sin^{-1}\left[\frac{t}{l_2}\frac{\pi}{2}\frac{\lambda g_0}{\lambda g}\left(1 + \frac{4l_2}{b_2\pi^2(\lambda g/\lambda g_0 - 1)}\right)^{1/2}\right]$$

$$\approx 2 \sin^{-1}\left[\frac{t}{l_2}\left(\frac{\pi^2}{4} + \frac{l_2}{b_2(\lambda g/\lambda g_0 - 1)}\right)^{1/2}\right]. \quad (21)$$

Returning to (9), we have

$$Z_I \approx jZ \cot \beta t = \frac{j2Zl_2}{\pi t}\frac{\lambda g}{\lambda g_0}. \quad (22)$$

Substituting these approximate expressions into (18), we have

$$L = 10 \log\left[1 - \frac{(Z_I - Y_I)^2}{4}\sinh^2(2n \sinh^{-1} P)\right] \quad (23)$$

where

$$P = \frac{t}{l_2}\left(\frac{l_2}{b_2(1 - \lambda g/\lambda g_0)} - \frac{\pi^2}{4}\right)^{1/2}. \quad (24)$$

For $\lambda g < \lambda g_0$, this expression approximates to

$$L \approx 10 \log\left[-\frac{(Z_I - Y_I)^2}{16}[(1 + P^2)^{1/2} + P]^{4n}\right]$$

$$= 40n \log[(1 + P^2)^{1/2} + P] + 20 \log\left[\frac{j(Z_I - Y_I)}{4}\right]. \quad (25)$$

Since l_2 is not normally much greater than $10t$, the contribution of the second term is not significantly greater than the minimum value of 6 dB. Therefore,

$$L + 6 > 40n \log[(1 + P^2)^{1/2} + P]. \quad (26)$$

If L_s is the minimum specified attenuation level and the frequencies of the stopband are at the guide wavelengths λg_1 and λg_2, then

$$\lambda g_0 = \lambda g_1$$

and

$$L_s + 6 < 40n \log[(1 + P_0^2)^{1/2} + P_0] \quad (27)$$

where

$$P_0 = \frac{t}{l_2}\left(\frac{l_2}{b_2(1 - \lambda g_2/\lambda g_1)} - \frac{\pi^2}{4}\right)^{1/2}. \quad (28)$$

For a good passband, this waveguide section must be imaged matched. Thus, from (15), for $Z_1 = 1$, we have

$$\frac{b_0}{b_2} = (1 + K^2)^{1/2} + K$$

where

$$K = \frac{l_2 A}{2b_0}. \quad (29)$$

Thus the complete design is given from λg_1, λg_2, L_s, and the frequency of the main passband, by the following.

1) l_2 is determined from λg_1.
2) A is obtained from l_2 and main passband frequency.
3) b_2 is determined from (29).
4) From (27) and (28), n is derived as

$$n > \frac{L_s + 6}{40 \log[(1 + P_0^2)^{1/2} + P_0]}$$

where

$$P_0 = \frac{t}{l_2}\left(\frac{l_2}{b(1 - \lambda g_2/\lambda g_1)} - \frac{\pi^2}{4}\right)^{1/2}. \quad (30)$$

Example: A waveguide filter is required to operate in WR137 and provide 30-dB attenuation from 9.8 to 10.6 GHz, while providing a good match VSWR < 1.1:1 from 6.4 to 6.9 GHz.

For the lower bandstop frequency, the effective length of the stubs is 0.36 in. Using 0.05-in plates, the true length will be 0.31 in. In the passband $A \approx 1.3$ and therefore, from (29), $b \approx 0.45$ in. Thus the sizes of the rectangular apertures in the two types of plates are 1.37 × 0.45 in and 1.37 × 1.07 in, both types being 0.05 in thick.

To determine n we first obtain $P_0 = 0.38$ and hence

$$n > \frac{1}{\log(1.45)}.$$

Therefore, $n = 7$ requires nine plates with the large aperture and eight with the small aperture, resulting in an overall length of 0.85 in.

Figs. 6 and 7 show the VSWR and insertion loss characteristics of the experimental filter, showing good agreement between theory and practice.

MULTIBAND BANDSTOP FILTERS

For multiband bandstop responses, it is only necessary to cascade the single filter section covering each of the prescribed stopbands and the same passband if all the stopbands are below the cutoff frequencies of the higher ordered modes in the main guide. However, if harmonic bands are to be attenuated, this will not be the case and higher ordered mode propagation must be considered.

In order to attenuate modes with field variations in the

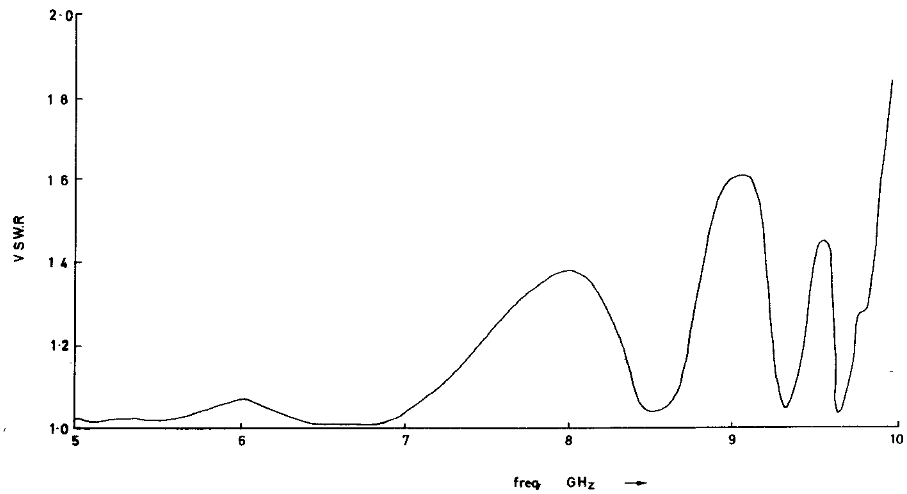

Fig. 6. VSWR of single stopband filter.

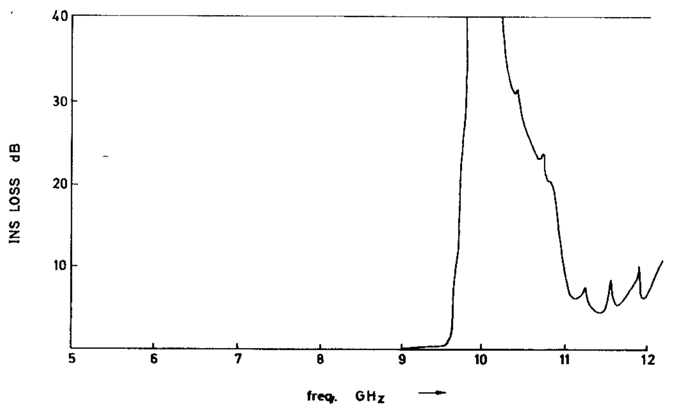

Fig. 7. Insertion loss of single stopband filter.

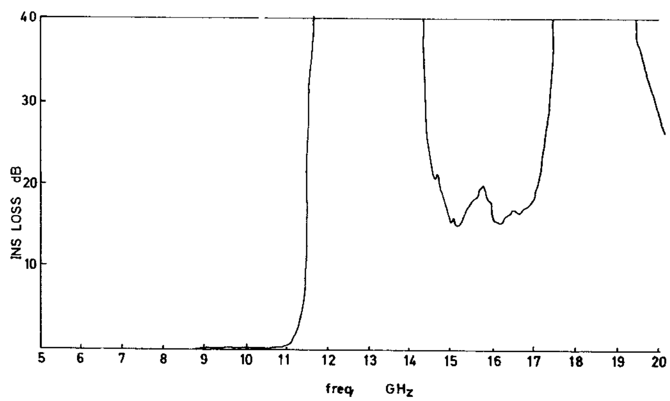

Fig. 8. Insertion loss of two-band harmonic rejection filter.

height dimension, at least one section of the filter should be of reduced height to constrain these modes to be evanescent. Assuming symmetry is maintained in the filter, the only modes which are allowed to propagate in the frequency range of interest may therefore be restricted to the $H_{0,2m-1}$ modes. Thus additional filter sections may be required to attenuate these modes. However, the design procedure remains the same, apart from the assumption that the effective waveguide width for dominant H_{01} design is reduced by a factor $2m - 1$.

In certain cases, additional elements may not necessarily be required if stopbands are to be incorporated below the onset of higher ordered modes as well as above. This may be readily illustrated by the following design example.

Example: Construct a waveguide filter in WR137 to provide a VSWR $< 1.1{:}1$ from 5.9 to 6.4 GHz, with an attenuation $>$ 40 dB, from 11.8 to 12.8 GHz and 17.7 to 19.2 GHz.

The filter is initially designed to provide the required attenuation of the H_{01} mode in the two bands of interest, but their effective impedance in the main passband are allowed to be variables. The filter is constructed by sandwiching the section attenuating the second harmonic and of effective image impedance Z_2 between two sections of the filter attenuating the first harmonic of effective image impedance Z_1. Z_2 is then chosen to be sufficiently large to attenuate modes with height variations in the second-harmonic band (i.e., the effective height $<$ 0.3 in). In the case of the H_{03} mode, the filter section providing attenuation to the H_{01} mode in the first-harmonic band will also attenuate the H_{03} in the second-harmonic band. However, additional elements will be necessary to provide the required level of attenuation. These additional plates may be chosen such that the sections in the filter are a quarter of a wavelength long in the main passband using the electrical length implied by (11). Thus Z_1 may then be finally chosen such that $Z_1 = (Z_2 Z_0)^{1/2}$ in the main passband, thus providing transfer matching into the reduced size waveguide.

The measured insertion loss characteristic of the designed filter is shown in Fig. 8. It should be noted that this particular filter is only 2 1/4 in long, and the VSWR $< 1.1{:}1$ from 5.9 to 6.4 GHz.

BANDPASS FILTERS

If bandpass channel filters are used in a system which also requires rejection of prescribed bands such as harmonic bands, then it may be advantageous to use the waveguide sandwich filter element as the basic resonant section in the design in place of the uniform waveguide normally used. For the direct-coupled cavity design [3], additional plates with the inductive coupling apertures are located at approximately half wavelength intervals in the effective guide wavelength, and consequently are readily incorporated into the laminar construction. The design procedure is similar to the previous cases where the sections are designed to meet the stopband requirements and then the effective impedances and wavelengths are used to obtain the values of the required inductive coupling apertures.

DISPERSIVE DELAY LINES

Using a large number of elements and operating below the resonant frequency of the series stubs, the waveguide sandwich filter provides a low attenuation with an effective phase constant

$$2n \sin^{-1}\left[t\beta\left(1+\frac{l_2 A}{b_2}\right)^{1/2}\right]. \tag{31}$$

As A becomes large, this phase variation tends towards a logarithmic variation (i.e., hyperbolic group delay) and consequently may be used to compress pulses in Doppler invariant pulse compression radar systems [4].

CONCLUSIONS

A new class of waveguide filter components has been described based on the basic waveguide sandwich filter section. The laminar construction using thin plates whose apertures may be formed using a punching process results in a very inexpensive device, particularly in the harmonic rejection application where tuning is not necessary. Experimental results have been presented which agree closely to predicted response behavior for the single and double stopband requirements. Additionally, these filters are significantly shorter than conventional bandstop filters and possess the advantage of being readily incorporated into existing systems with the minimum amount of effort, particularly since they may be mounted directly between normal waveguide flanges.

There are many possible methods of incorporating these waveguide sections into existing component design. For example, in a conventional harmonic rejection filter requirement, additional attenuation may be necessary in a particular band. If a corrugated waveguide design were being used, then some of the elements in the filter could be modified to sandwich sections, thus meeting the additional requirement without increasing the overall length. Further possible applications in the bandpass channel filter area have been mentioned, and finally, the possible application as a dispersive delay line has been cited.

ACKNOWLEDGMENT

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